

Mania?

7.4.57

Rationalize the denominator. Assume that all expressions under radicals represent positive numbers.

$$\begin{aligned} \sqrt{\frac{441ab^4}{1372a^5b}} &= \frac{\sqrt{441ab^4}}{\sqrt{1372a^5b}} = \frac{\sqrt{4 \cdot 49 \cdot 3 \cdot a^4 b^4}}{\sqrt{4 \cdot 343 \cdot 4 \cdot b}} \\ &= \frac{2 \sqrt{49 \cdot 3 \cdot a^4 b^4}}{2 \sqrt{343 \cdot 4 \cdot b}} = \frac{2 \sqrt{147} a^2 b^2}{2 \sqrt{1372} b} \\ &= \frac{\sqrt{147} a^2 b^2}{\sqrt{1372} b} \end{aligned}$$

Handwritten calculations for prime factorization and simplification:

$$\begin{aligned} 1372 &= 2 \cdot 686 = 2 \cdot 2 \cdot 343 = 2^3 \cdot 7^3 \\ 441 &= 9 \cdot 49 = 3^2 \cdot 7^2 \end{aligned}$$

fast way

$$\begin{aligned} &\frac{\sqrt{441ab^4}}{\sqrt{1372a^5b}} \\ &= \frac{\sqrt{49 \cdot 9 \cdot ab^4}}{\sqrt{49 \cdot 4 \cdot 7 \cdot a^4 \cdot ab}} \\ &= \frac{\sqrt{7^2 \cdot 3^2 \cdot ab^4}}{\sqrt{7^2 \cdot 2^2 \cdot 7 \cdot a^4 \cdot ab}} \\ &= \frac{7 \cdot 3 \cdot b^2 \sqrt{a}}{7 \cdot 2 \cdot a^2 \sqrt{7ab}} \cdot \frac{\sqrt{7ab}}{\sqrt{7ab}} \\ &= \frac{3b^2 \sqrt{7a^2 b}}{2a^2 \cdot 7ab} = \frac{3ab^2 \sqrt{7b}}{14a^2 b} \end{aligned}$$

$$\boxed{\frac{3b\sqrt{7b}}{14a^2}}$$

Handwritten prime factorization of 1372:

$$\begin{array}{r} 49 \\ 28 \\ \hline 1372 \end{array}$$

$$\begin{aligned} &= \frac{3b^2 \sqrt{49a}}{2a^2 \sqrt{343ab}} \cdot \frac{\sqrt{343ba}}{\sqrt{343ba}} = \frac{3b^2 \sqrt{49a \cdot 343ba}}{2a^2 \cdot 343ba} \\ &= \frac{3b \sqrt{16807ab}}{2a^2 \cdot 343a} = \frac{3b \sqrt{7^2 \cdot 7 \cdot 343b}}{2a^2 \cdot 343a} \end{aligned}$$

$$\begin{aligned} &= \frac{21ab \sqrt{343ab}}{2a^2 \cdot 49 \cdot 7a} = \frac{3 \cdot 7ab \sqrt{49 \cdot 7b}}{7^3 \cdot a \cdot 2a^2} \\ &= \frac{3 \cdot 7ab \cdot 7 \sqrt{7b}}{7^3 \cdot a \cdot 2a^2} \\ &= \frac{3b\sqrt{7b}}{7 \cdot 2a^2} \end{aligned}$$

$$\boxed{\frac{3b\sqrt{7b}}{14a^2}}$$