When you have completed this activity, go to Status Check.

Pre-Algebra A
Unit 2

Student Activity: Finding Factors and Prime Factors

Name _____ **Date**_____

Objective

In this activity, you will find the factors and the prime factors of whole numbers.

Materials

Ruler

Vocabulary

factor: a whole number that can be multiplied by another whole number to form a product (Example: 2 and 3 are factors of 6 because $2 \times 3 = 6$)

prime number: a whole number that has only 1 and the number itself as factors (Examples: 2, 5, 11)

composite number: a whole number that has factors other than 1 and the number itself (Example: 9)

prime factorization: a list of prime factors that form a certain product (Example: $3 \times 5 \times 5 = 75$)

Warm-Up

The numbers 2, 3, and 5 are prime numbers. You can't divide them by any whole numbers except 1 and themselves. That means the multiplication statements below are the only possible ones with these prime numbers as the product. (Remember, 1 is not considered a prime or a composite number.)

$1 \times 2 = 2$ or $2 \times 1 = 2$ $\quad$ $1 \times 3 = 3$ or $3 \times 1 = 3$ $\quad$ $1 \times 5 = 5$ or $5 \times 1 = 5$

We multiplied these prime numbers by each other. The products are the composite numbers in the column on the right. Match the multiplication problem with the products by writing them in the blanks.

$5 \times 5 \times 2 =$ _____	18
$5 \times 5 \times 3 =$ _____	60
$3 \times 3 \times 5 =$ _____	50
$5 \times 2 \times 2 \times 3 =$ _____	15
$2 \times 5 \times 3 =$ _____	45
$2 \times 3 \times 3 =$ _____	30
$5 \times 3 =$ _____	20
$2 \times 5 \times 2 =$ _____	75

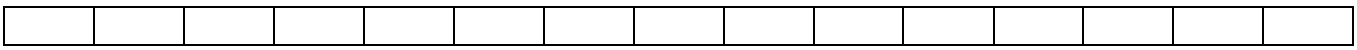
You've just completed the prime factorization of these composite numbers.

Study

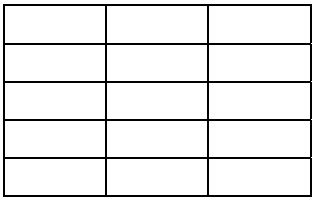
Arrays and Factors

These arrays help to show the factors of 15. You can arrange 15 rectangles into the arrays shown. Fill in the missing numbers that go with each array.

_____ × _____ = 15



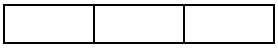
_____ × _____ = 15



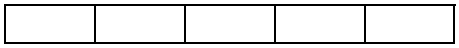
Now use the arrays to list all the factors of 15: _____ _____ _____ _____

What if you start with only 3 or with only 5 rectangles? Can you arrange 3 or 5 rectangles in other ways and have the same number of columns in each row? _____

$1 \times 3 = 3$



$1 \times 5 = 5$

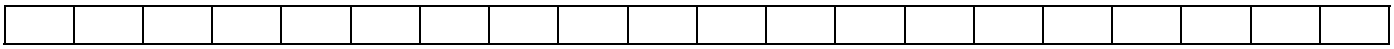


Explain why you can or cannot rearrange the rectangles.

List the factors of 3 and 5. Factors of 3: _____ Factors of 5: _____

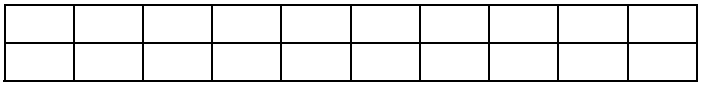
Now let's look at the number 20. We'll use arrays to find its factors.

$1 \times 20 = 20$



Fill in the missing numbers for this array of 20 rectangles.

_____ × _____ = 20



The number 20 has one more set of factors that isn't shown in the arrays above. Draw an array in the space below to show the missing set of factors. Then fill in the missing numbers. (You may want to use a ruler to draw the array.)

____ × ____ = 20

Now use the arrays to list all the factors of 20: _____

Dividing to Find Factors

You can also find the factors of a number without drawing arrays. Start with the smallest numbers and check if they are factors, and then work up to bigger numbers. Keep a list of the factors as you go.

Let's see how this works with 12.

You know 1 is a factor, and $1 \times 12 = 12$. So you'd put **1** and **12** on the list.

Check to see if 2 is a factor. It is because $2 \times 6 = 12$. So you add **2** and **6** to the list of factors.

Check to see if 3 is a factor. It is because $3 \times 4 = 12$. So you add **3** and **4** to the list of factors.

We already listed 4 as a factor, 5 is not a factor, and 6 is on the list already. If we check 7, 8, 9, 10, and 11 we'll find that they aren't factors either. We already have 12 on the list.

So the factors of 12 are **1, 2, 3, 4, 6,** and **12.**

Use the same process to find the factors of 54. First list the smaller numbers 54 is divisible by and the factor that number goes with. (Use as many of the blanks as you need.) Then list all the factors in the blank below.

Factors of 54:

_____×_____=54

_____×_____=54

_____×_____=54

_____×_____=54

_____×_____=54

_____×_____=54 All the factors of 54: _____

Prime Factors

Let's review prime numbers before we find the prime factors of composite numbers.

Here's a reminder of what *prime* and *composite* mean:

Prime number: a number that has only two factors, which are 1 and the number itself. This means that it is not divisible by any whole numbers except 1 and itself.

Composite number: a number that has more than two factors. This means that it is divisible by other numbers than 1 and itself.

Underline all the prime numbers in this list:

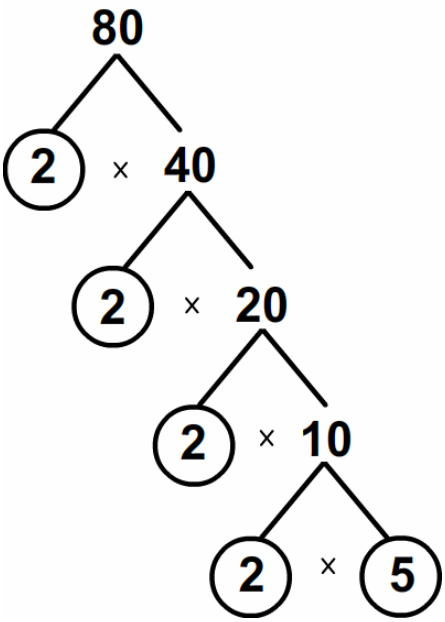
2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29

***Note about the number 1**:* The number 1 is not considered prime or composite. Because it has only one factor—itself—it's neither. That means it won't be included in a list of prime factors of a number.

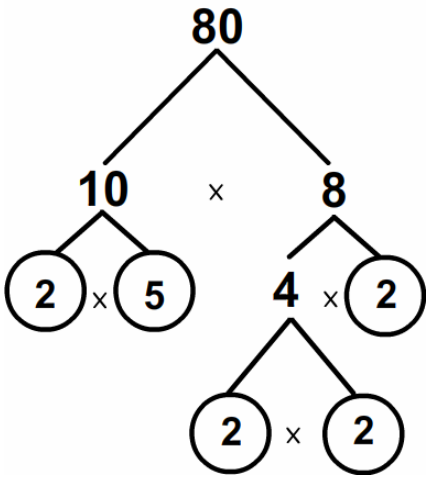
So, the factors of 6, for example, are 1, 2, 3, and 6. But the prime factors of 6 are just 2 and 3.

Factor Trees

One way to find prime factors is to use a factor tree. To factor the number 80, for example, we start dividing it until we end up with only prime numbers. Here are two ways to make a factor tree for 80.



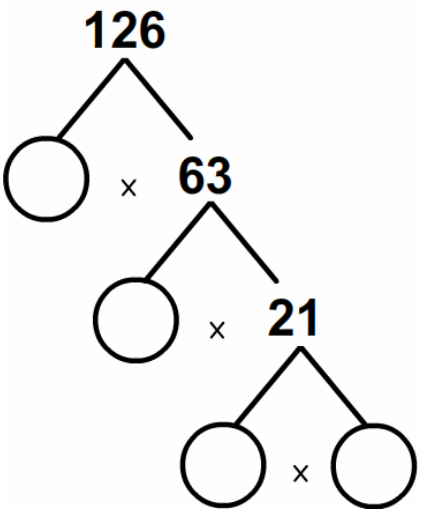
$2 \times 2 \times 2 \times 2 \times 5 = 80$

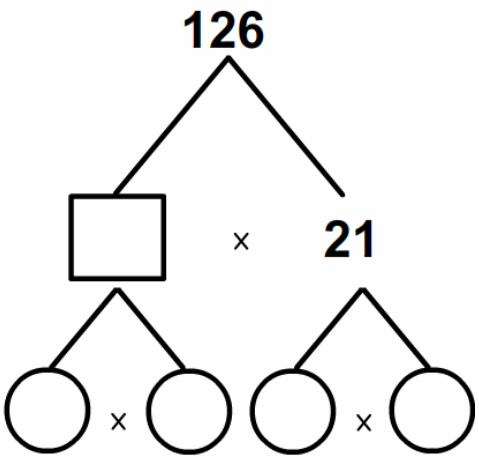


$2 \times 5 \times 2 \times 2 \times 2 = 80$

In the factor tree on the left, we divided 80 by the prime number 2 until we got the prime number 5. In the factor tree on the right, we divided by composite factors of 80 and factored down until each number was the product of prime numbers. We circled each prime number so that it is easier to list the prime factors from the factor tree.

Finish these factor trees for 126 by filling in the missing numbers. Then write the prime factorization for 126 in the blank below.





_____ = 126

Which factor tree did you find more useful for finding the prime factors of 126? Explain.

Dividing

This is another way to find the prime factors of a number. Instead of using a tree to show the division steps, you can divide a number repeatedly. Start with small prime numbers and see if they are factors of the number.

Let's find the prime factors of 84. We'll start with familiar small prime numbers like 2, 3, and 5.

Do you think 84 is divisible by 2? ____ Do you think it's divisible by 3? ____ By 5? ____

Let's start by dividing 84 by prime numbers so we can list it as a product of primes. Find solutions for the division problems below. Rewrite each solution in the first blank of the next line to continue factoring 84.

$84 \div 2 =$ ____

(That result) ____ $\div 2 =$ ____

(That result) ____ $\div 3 =$ ____

Your last result should be a prime number. It is also a factor of 84. Now, rewrite 84 as the product of each of the numbers you divided it by and your last result.

$84 =$ ____ $\times$ ____ $\times$ ____ $\times$ ____ This is the prime factorization of 84.

Note: There is no correct order to list prime factors. But it is conventional to list the smallest primes factors first and then the higher ones (be sure to include every factor, even if it occurs more than once). So the prime factorization of 18 should be shown as $2 \times 3 \times 3 = 18$ rather than $3 \times 2 \times 3 = 18$.

Practice

1. Make a factor tree for the numbers below. Then write the prime factorization of each number in the blank.

_____ = 24

_____ = 36

_____ = 70

_____ = 46

2. Which list shows only prime numbers?

   a) 2 5 7 11 13

   b) 2 5 7 9 11

   c) 3 4 7 8 11

   d) 2 5 6 7 11

3. List all the factors of these numbers.

   Factors of 24: _____

   Factors of 36: _____

   Factors of 70: _____

   Factors of 46: _____

Wrap-Up

Take the numbers 100 and 32. Draw a factor tree to write the prime factorization for one of the numbers, and use the division method to find the prime factorization for the other number.

Prime factorization of 100: (Method used for 100: _____)

Prime factorization of 32: (Method used for 32: _____)

Now list **all** the factors for both numbers.

Factors of 100: _____

Factors of 32: _____

Finally, describe how prime factorization can help you find all the factors of a number.